us than as they are developed in our subjectivity. The idea is as absurd as if we should say that the number three is a part of a group of three persons. Three is not in any way comparable with three persons.

Dr. Carus is very right when he says (p. 63):

"The problems concerning the foundations of geometry and of mathematics in general are by no means so definitely settled that one solution may be said to have acquired the concensus of the competent, and for this reason I feel that a little mutual charity is quite commendable."

Thus, if I may differ somewhat in opinion from Dr. Carus, I must openly beg his charity for advocating my own views against him. I may have been led to these discussions "by an enthusiasm as strong as the zeal of religious devotees which....has a humorous aspect," but I am of the firm belief that they will perchance "serve to widen the horizon of his views," although not endowed with the positive power of "reversing, antiquating or abolishing the assured accomplishment" of Dr. Carus.

With us it is *never* "strange that the nature of man's rationality is by no means universally recognized." It seems very natural that "opinions vary greatly concerning its foundation and its origin." We are quite satisfied with the coexistence of various different systems, and so we shall be always happy to receive varying criticisms.

YOSHIO MIKAMI.

OHARA IN KAZUSA, JAPAN, March, 2, 1910.

EDITORIAL COMMENT.

On a first perusal of Mr. Yoshio Mikami's criticism of my views concerning the foundations of geometry, I thought that no reply would be needed for any one who has read my main expositions of the problem, the article in question as well as my books *Kant's Prolegomena* and *The Foundations of Mathematics*. But I am anxious to let every criticism receive consideration, and so I take pleasure in publishing Mr. Mikami's remarks. Since, however, many of our readers have not read the writings under discussion, I will briefly point out why Mr. Mikami's arguments fail to apply to my position.

It is true enough that I propose to lay the foundation of geometry without having recourse to axioms. However I have not for that reason, as Mr. Mikami says, "unconsciously introduced an assumption or assumptions," but I build all the formal sciences upon the facts of our own existence. In doing so I simply follow the

genetic process of mathematical conceptions. Mathematical conceptions did not originate through assumptions or arbitrarily invented axioms but like the idea of numbers they are due to abstraction, and they originated naturally in the course of the evolution of the human mind at a certain period when man was ready for them.

We cannot construct anything from nothing. The idea of building mathematics on emptiness is unjustified, but I claim that the method as well as a field of action were procured together with its definite purpose at the time of its origin by the needs of the situation. And it is rather strange that this simplest method of investigating the genesis of mathematics has not yet been attempted for laying its philosophical foundation. Here Mr. Mikami has utterly failed to understand my position, and I wonder that he criticised me so boldly while he is unfamiliar with the most important arguments which I have tried to impress upon my readers.

The domain of mathematics is a field of anyness, and so long as Mr. Mikami omits the very mention of this conception, he will be incapable of understanding, let alone criticizing, my position. The very word "anyness" throws a flood of light upon the problem and helps us to solve it. As soon as man learns to speak, he can discriminate between concrete and abstract things. He generalizes and speaks of qualities which do not exist by themselves, and when he comes to generalize the purely formal aspects of experience he creates notions which do not apply to one concrete object alone but to any object, and thus acquire a universal significance. This possibility of thinking in terms of anyness is the foundation of all science and especially of the formal sciences.

Bodily forms are concrete, but pure forms are of an abstract nature; they are mental constructions. Pure form is purely relational; it is a matter of arrangement, either succession or juxtaposition, and contains nothing which can be expressed in terms either of matter or energy.

The idea of form has been ultimately derived from experience, for there is nothing in the world of our senses which is not somehow endowed with form, and he who speaks of objects as being devoid of form denies the most obvious facts of our experience.

Experience furnishes the data of all our knowledge, and these data can be analyzed into the sense elements of feelings and their forms. The generalization of the idea of form leads to one very peculiar result, which is, that the constructions we make apply generally for any case of the same kind. The reason is simple enough.

Form is the most abstract quality which is common to all things, and so we characterize the purely formal as anyness. But there is another point to be noted. When dealing with sense experience we have always before us concrete and isolated cases, but in making constructions of pure form we can exhaust all possibilities and so we can be systematic. Instead of observing isolated cases we can formulate a general law, which means a description of the essential features of all possible cases. Here lies the significance of the purely formal sciences, and this is the reason why the nature of form is the fundamental problem of science and philosophy. The purely formal sciences furnish us with a general scheme excluding impossibilities, and are of such a nature as to permit us to arrange all possible cases systematically. If formal thought were not capable of furnishing such *a priori* systems, science would be impossible.

We have seen that the idea of anyness originated by abstraction, by dropping all features of concreteness, and we know that primitive man began purely formal operations, such as counting, by creating a system of reference in units. He counted heads of cattle on his fingers and he interrelated the objects to be counted with his names of units or with some mnemotechnic help which served him as an abacus. We cannot doubt that man originally used his fingers as a system of reference, though the essential things were not his concrete fingers but the idea of units which the fingers represented.

Accordingly arithmetic and in the same way geometry did not originate from nothing, but through abstraction by omitting those features of experience which at the moment were not wanted for the purpose of understanding a certain situation.

The mode of creating such systems of anyness is due to man's mental activity from which, however, anything concrete, be it matter or energy, has been excluded. In arithmetic this pure activity is a progress from point to point, thereby creating discrete units; in geometry, however, we trace continuous paths of our motion called lines. We start with our ability to do certain things; we limit our activity to the abstract field of anyness and then we proceed to make constructions of pure form. No assumptions nor axioms are needed, except the principle of consistency. And we may create the conditions as we please. We may build up a system of numerals or the plane of Euclidean geometry. We may think of any lines of the same size as equal, or we may also consider direction and treat lines as vectors.

In one sense anyness is nothing. It is a state of being devoid

of anything definite and concrete, but it is not, for that reason, absolutely nothing. The field of anyness possesses definite positive qualities, among which most significant is the quality of the absence of all peculiarity, which means that the same action taken now and here is the same as if taken at any other time or in any other place. The field of operation is throughout the same, and so constructions are different only if they have been made different. In arithmetic a unit is a unit whenever or wherever it is posited, and in geometry progress can be made in any direction and without any limitation, but the same figure will always be the same.

Note that the principle of action without further limitation involves the highly important concept of infinitude. The idea of a progress from unit to unit implies that wherever I stop I might continue, and there is a possibility of progressing to further units beyond any stopping place. It is strange that the idea of infinitude has been a stumbling block to the minds of many thinkers, profound as well as shallow, mystics as well as scientists, but I wish to say here that from my standpoint infinitude is the simpler concept, and finiteness a more complicated idea. The field of action without further limitation is a primitive idea in the fundamentals of mathematics, and so any kind of field of *a priori* activity will be infinite unless by a special assumption a limit is imposed upon the activity with which we start. However, we do not get rid of infinitude, even if we limit our field of operation and make it finite in one way or another, because the very idea of a limit is a boundary which implies a *cis* and a *trans.* If there is a boundary we postulate a beyond. Mr. Mikami does not recognize the logical necessity of this statement, for he speaks of spherical space, and complains that I introduce into my notion of spherical space the idea of Euclidean space with its infinitely straight line. But such is not the case. I only introduce a logical principle, for even if we have a spherical space we would have to determine the radius of the sphere, and here again we would have the choice of a radius from the infinitely small to the infinitely great, and a sphere of the radius of the infinitely great would again restore infinitude to its proper birthplace. If, however, we assume a spherical space of a definite radius, we have a very concrete case, and have left the field of anyness, which according to my conception of the foundations of mathematics is the fundamental idea without which we will be bewildered by a tangle.

Not having familiarized himself with my views of anyness, Mr. Mikami does not understand that our space-conception may

be ultimately based on experience, while in spite of it the construction of mathematical space is *a priori* and purely formal. He sees a contradiction in the two statements, "without motion no space-conception" and "pure mathematics does not depend upon the senses." Mr. Mikami declares that the former statement is tantamount to saying that "our notion of space is ultimately based upon our senses." Does he deny that we can make abstractions? I grant that in reality we can not produce "whiteness" as a thing by itself, or "motion-in-itself," a change of place without moving objects and devoid of energy. But in thought we can create such abstract ideas, and I claim that the whole field of mathematics is such an abstract conception which does not exist in objective reality; it is purely mental. Being a construction which purposely omits everything concrete, mathematics is devoid of sense elements. Experience, as I understand the word, consists of sense perceptions, and sense perceptions contain both elements, the sensual and the formal. By omitting the sensual we retain the idea of pure form, and so all systems of pure form are products of the mind, and are constructed by means of abstractions ultimately derived from experience.

Kant's transcendentalism is based on the argument that mathematical constructions are *a priori,* and so, Kant claims, they can not have been deduced from experience. He insists that they are the condition of all experiences, for experience becomes only possible by relying upon the purely formal sciences, including pure natural science which is based on the conception of causality. I can not look for causes or the effects of causes, unless I have in my mind the idea of the law of causation. These conditions of all experience Kant calls transcendental, and transcendental ideas, such as logic, arithmetic, geometry, or in a word reason, as well as the conceptions of time and space form the constitution of the human mind; but how mind originates Kant has never investigated.

I find fault with Kant's use of the term "experience" which he mostly restricts to the idea of sense experience but sometimes employs in the broader meaning of sense experience as guided by logic and other principles of formal thought. Mathematics has nothing to do with experience in the narrower sense, but the means of its construction have been derived by abstraction from experience in the broader sense. Accordingly my propositions do not involve a contradiction as Kantians would be inclined to think and as Mr. Mikami actually declares.

There is another point on which my view differs from that of

Kant. It is what he calls idealism, but which is truly subjectivism.

The domain of the mind is the realm of ideas, and so Kant concludes that time and space and reason (or in a word all branches of formal thought) are ideal, and he uses the term in contrast to real or objective. In truth he identifies the term "ideal" with "subjective," and thus he claims that forms appertain to the mind and not to the objective world. Here lies the fallacy of Kant. We must consider that there is no subject in itself. Every thinking subject is a concrete and real body moving about as an object in the objective world. A thinker considered as a subject is only the inner aspect of an objective personality, and this objective personality is as much a part and parcel of the objective world as any other object. The experience of a subject is due to the objective contact of a thinking being, and this contact is experienced, not in pure subjectivity but by its bodily and objective sense organs.

The experiences of a thinker are first of all part and parcel of his objective body as it moves and is moved about, as it pushes and is pushed, as it is exposed to objective contact, mechanical as well as chemical or electric, and otherwise in its relation in the objective world. Form accordingly, with its quality of relationship, of juxtaposition, of difference of structure, etc., is a feature of the objective world and the idea of form is its representation in the domain of subjectivity. Accordingly the evidence that form is purely subjective is not forthcoming and stands in contradiction to what we know about the nature of form. If form were purely subjective, we would be compelled to deny objectivity altogether.

The abstractions from which the purely formal sciences have been created have been derived from experience, and since at the same time the formal sciences serve a practical purpose, we must assume that the objective world contains features which somehow correspond to its fundamental conceptions. This is certainly borne out by experience, for the formal sciences are the most indispensable part of our cognition. Without them man would not be a rational being.

We have repeatedly insisted upon the truth that all mathematical sciences, logic as well as arithmetic, are ideal in the sense that they are mental constructions. There are no logarithms in the objective world, but only in our mind, and the same is true of our idea of purely formal motion. There are no numbers running about in the starry heavens nor in the world of chemical atoms. Nevertheless the objective world is so constructed that by counting and measuring

we can acquire an insight into its constitution. We can determine magnitudes, distances and other properties of objects, and that is all that is needed.

Human reason exists as reason only in the human brain, but there are features in the objective world which make it possible that the theorems of reason assist us in comprehending the conditions of things. This objective counterpart of human reason has been characterized as the cosmic world order. The Germans call it *Gesetzmässigkeit,* a word which we have translated by "lawdom," meaning a state which admits of a description in so-called laws of nature. Mathematics more than any other science, helps us to understand this lawdom of the objective world, and although mathematical conceptions are purely mental, although there are no trigonometrical ideas, no sines nor cosines, no algebraic formulas extant in the objective world, the theorems of mathematics, being constructed in the field of anyness, help us to understand any analogous products; and also to render possible thereby a comprehension of this real world of ours.

ON THE MAGIC CIRCLE.

In the author's article on "Mediæval Occultism" (*The Monist,* XVIII, 510) a suggestion was made to the effect that the magic circle which forms an integral part of all thaumaturgic ritual served to define or limit the magical environment. Further consideration on this matter combined with a study of Buddhist and Chinese occultism has led the author to extend the use of this circle to a considerable extent.

It has long been recognized among anthropologists that temples as the residences of supernal powers represent in miniature the universe, and it is not difficult to show that the circle, with two perpendicular diameters oriented, is also a very widely used symbol for the universe, so that the magus operates as it were within a universe of his own creation. This then is the thesis of the present article, and it may be defined more generally as follows:

"The magic circle is an essential feature of magical operations, and expresses symbolically the universe. Within this circle the magus by the processes of ritual evokes supernatural powers (as he conceives them to be) with a space relation to the corresponding positions in the physical universe and the ideal universe of occult philosophy.